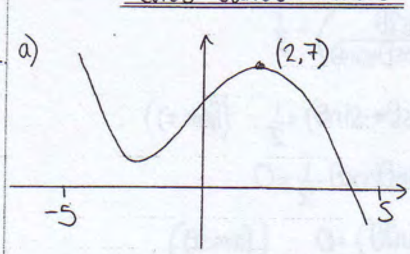
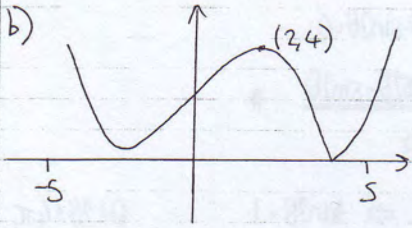
(1)

Core3 - Jan06 Solutions

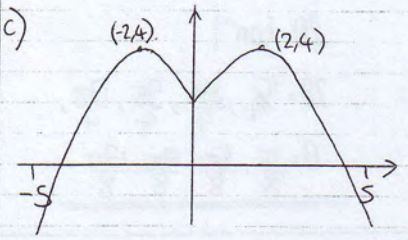
1. a)



b)



c)



2. $\frac{2x^2+3x}{(2x+3)(x-2)} - \frac{6}{x^2-x-2}$

$= \frac{x\cancel{(2x+3)}}{\cancel{(2x+3)}(x-2)} - \frac{6}{(x-2)(x+1)}$

(2)

$= \frac{x(x+1)}{(x+1)(x-2)} - \frac{6}{(x+1)(x-2)}$

$= \frac{x^2+x-6}{(x+1)(x-2)}$

$= \frac{(x-2)(x+3)}{(x+1)(x-2)} = \frac{x+3}{x+1}$

3 $y = \ln(\frac{1}{3}x)$ when $x=3$, $y = \ln(\frac{1}{3}\times 3) = \ln(1) = 0$

$\frac{dy}{dx} = \frac{1}{x}$ $\therefore$ $\left.\frac{dy}{dx}\right|_{x=3} = \frac{1}{3}$

$\therefore$ Grad of normal $= -3$

When $x=3$ $y=0$ $\therefore$ $y-0 = -3(x-3)$

$y = -3x+9$

4 a) I) $y = \underset{u}{x^2}\underset{v}{e^{3x+2}}$ $u = x^2$ $u' = 2x$ $v = e^{3x+2}$ $v' = 3e^{3x+2}$

$\frac{dy}{dx} = 3x^2e^{3x+2} + 2xe^{3x+2}$

II) $y = \frac{\cos(2x^3)^{u}}{3x_{v}}$ $u = \cos(2x^3)$ $u' = -6x^2\sin$

$v = 3x$ $v' = 3$

(3)

$\frac{dy}{dx} = \frac{3x \times -6x^2\sin(2x^3) - 3\cos(2x^3)}{9x^2}$

$= \frac{-6x^3\sin(2x^3) - \cos(2x^3)}{3x^2}$

b). $x = 4\sin(2y+6)$

$\frac{dx}{dy} = 8\cos(2y+6)$

If $x = 4\sin(2y+6)$

$x^2 = 16\sin^2(2y+6)$

$x^2 = 1 - 16\cos^2(2y+6)$

$\cos^2(2y+6) = \frac{1-x^2}{16}$

$\cos(2y+6) = \frac{\sqrt{1-x^2}}{4}$

$\therefore \frac{dx}{dy} = 8\frac{\sqrt{1-x^2}}{4}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{1-x^2}}$

(4)

5 a) $2x^3 - x - 4 = 0$

$x^3 = \frac{x+4}{2}$

$x^2 = \frac{x+4}{2x}$ $\therefore$ $x^2 = \frac{1}{2} + \frac{2}{x}$ $x = \sqrt{\frac{1}{2} + \frac{2}{x}}$ #

b) $x_1 = \sqrt{\frac{2}{1.35} + \frac{1}{2}} = 1.41$

$x_2 = \sqrt{\frac{2}{1.4077} + \frac{1}{2}} = 1.39$

$x_3 = \sqrt{\frac{2}{1.3859} + \frac{1}{2}} = 1.39$

c) $f(1.3915) = 2(1.3915)^3 - 1.3915 - 4 = -0.0029$

$f(1.3925) = 2(1.3925)^3 - 1.3925 - 4 = 0.0078$

change of sign indicate root and $\therefore$ $\alpha = 1.392$ to 3dp. #

6 a) $R\cos(x+\alpha) = 12\cos x - 4\sin x$

$R\cos x\cos\alpha - R\sin x\sin\alpha = 12\cos x - 4\sin x$

$R\cos\alpha = 12$ $R\sin\alpha = 4$

$\tan\alpha = \frac{4}{12} = \frac{1}{3}$ $\Rightarrow \alpha = 18.4°$

⑤

$R^2 = 12^2 + 4^2 = 160$

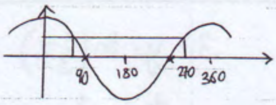
$\underline{R = 4\sqrt{10}}$

b) $4\sqrt{10}\cos(x+18.4^\circ) = 7 \qquad 0 \le x \le 360$

$\cos(x+18.4^\circ) = \frac{7}{4\sqrt{10}} \qquad 18.4^\circ \le x+18.4^\circ \le 378.4^\circ$

$x+18.4^\circ = 56.4^\circ, 303.6^\circ$

$\underline{x = 38.0^\circ, 285.2^\circ}$



c) I) min value $-4\sqrt{10}$

II) When $x+18.4^\circ = 180^\circ \Rightarrow \underline{x = 161.6^\circ}$

7a) I) $\frac{\cos 2x}{\cos x+\sin x} = \frac{\cos^2 x-\sin^2 x}{\cos x+\sin x} = \frac{(\cos x-\sin x)(\cos x+\sin x)}{\cos x+\sin x}$

$= \underline{\cos x - \sin x}$ #

II) $\frac{1}{2}(\cos 2x - \sin 2x) = \frac{1}{2}(\cos^2 x - \sin^2 x - 2\sin x\cos x)$

$= \frac{1}{2}(\cos^2 x - (1-\cos^2 x) - 2\sin x\cos x)$

$= \frac{1}{2}(2\cos^2 x - 1 - 2\sin x\cos x)$

$= \underline{\cos^2 x - \tfrac{1}{2} - \sin x\cos x}$ #

⑥

b) $\cos\theta\left(\frac{\cos 2\theta}{\cos\theta+\sin\theta}\right) = \frac{1}{2}$

$\cos\theta(\cos\theta - \sin\theta) = \frac{1}{2}$ (from I)

$\cos^2\theta - \sin\theta\cos\theta - \frac{1}{2} = 0$

$\frac{1}{2}(\cos 2\theta - \sin 2\theta) = 0$ (from II)

$\cos 2\theta - \sin 2\theta = 0$

$\underline{\cos 2\theta = \sin 2\theta}$ #

c) $\sin 2\theta = \cos 2\theta$

$\frac{\sin 2\theta}{\cos 2\theta} = 1 \Rightarrow \tan 2\theta = 1 \qquad 0 \le 2\theta \le 4\pi$

$2\theta = \tan^{-1} 1$

$2\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4},$

$\underline{\theta = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}}$

9 a) $g(f(x)) = e^{2(2x+\ln 2)}$

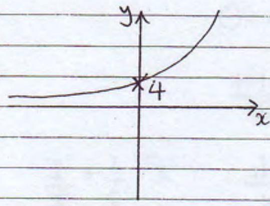
$= e^{4x+2\ln 2}$

$= e^{4x}e^{2\ln 2}$

$= e^{4x}e^{\ln 4} = \underline{4e^{4x}}$ #

⑦

b)



c) $gf(x) > 0$

d) $\frac{d}{dx}(4e^{4x}) = 3$

$16e^{4x} = 3$

$e^{4x} = 3/16$

$4x = \ln(3/16)$

$x = \frac{1}{4}\ln(3/16) = -0.418$ (3sf)